



PANJAB UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science Honours in Applied Mathematics	
QUALIFICATION CODE: 08BHAMS	LEVEL: 8
COURSE CODE: PDE801S	COURSE NAME: Partial Differential Equations
SESSION: JUNE 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 70

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	PROF. D. O. MAKINDE

INSTRUCTIONS
<ol style="list-style-type: none">1. Answer ALL the questions in the booklet provided.2. Show clearly all the steps used in the calculations.3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Question 1(20 marks)

- 1.1 Obtain the solution of the given PDE by direct integration method if

$$\frac{\partial^2 u}{\partial x^2} = 24x^2 (t - 2) \quad \text{when}$$

$$x = 0, \quad u = e^{2t}, \quad \text{and} \quad \frac{\partial u}{\partial x} = 4t \quad \text{are functions of } t. \quad [9]$$

- 1.2 Show that $u = f(x)g(y)$ where f and g are arbitrary twice differentiable functions satisfies $uu_{xy} - u_x u_y = 0$ [11]

Question 2 (17 marks)

Use the separable variables method to solve the inhomogeneous problem

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2 \quad \text{using the initial condition that}$$

$$u(x, 0) = 3 \exp\left(\frac{x^2}{4}\right). \quad [17]$$

Question 3 (13 marks)

Consider a stretched string of length l fixed at both end points.

Determine from basic principles the equation of motion which characterizes the position $u(x, t)$ of the string at time t after an initial disturbance.

Assume there is only a pure transverse vibration of the string. [13]

Question 4 (20 marks)

Obtain the steady state temperature distribution in a cube as described by the Laplace's equation

$$\nabla^2 u = u_{xx} + u_{yy} + u_{zz} = 0 \quad \text{for the intervals}$$

$$0 < x < \pi, \quad 0 < y < \pi \quad \text{and} \quad 0 < z < \pi$$

when the faces are kept at zero degree temperature except the face $z = 0$ and

$$u(0, y, z) = u(\pi, y, z) = 0$$

$$u(x, 0, z) = u(x, \pi, z) = 0$$

$$u(x, y, \pi) = 0, \quad u(x, y, 0) = f(x, y)$$

as the initial boundary conditions. [20]

END OF EXAMINATION